Stability of Pathological Detonations

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Abstract

Although the governing equations for detonation waves admit steady, one-dimensional solutions, experiments show that detonations usually have a three-dimensional, time-dependent structure. The underlying steady wave is thus unstable to perturbations. While much attention, both analytical and numerical, has been given to the stability of detonations where the unsupported detonation wave is the Chapman-Jouguet detonation, very little attention has been given to systems where the unsupported wave is of the pathological (or eigenvalue) type (which is probably due to the difficulty of determining even the steady, one-dimensional wave). We investigate the stability of such pathological waves, including both a linear stability analysis and time-dependent numerics.

Introduction

Pathological detonations can occur in systems which have endothermic or dissipative effects (such as endothermic stages in the reactions, mole changes during the reaction, slight curvature of the detonation front, gas with relaxational degrees of freedom, inclusion of transport effects, more than one reversible reaction, etc.). An example, in an astrophysical setting, are nuclear detonation waves in the stellar material of white dwarf stars (which are thought to be the progenitors of some supernovae events), in which the detonation is of the pathological type, due to an endothermic final stage of the reactions.

When such detonations occur, the Chapman-Jouguet detonation is unobtainable. Indeed, pathological detonations travel at the unsupported detonation speed for the system, which is faster than the corresponding CJ speed, and have an internal (frozen) sonic point where the thermicity is zero. Downstream of the sonic point there are two possible reaction zone structures, a subsonic (i.e. strong) branch, which corresponds to a supported pathological detonation, and a supersonic (i.e. weak) branch which corresponds to the unsupported detonation (see figure 1 for example). Overdriven detonations for such systems have an internal minimum in the pressure, and the unsupported pathological detonation wave structure is quite different from the supported wave structure.

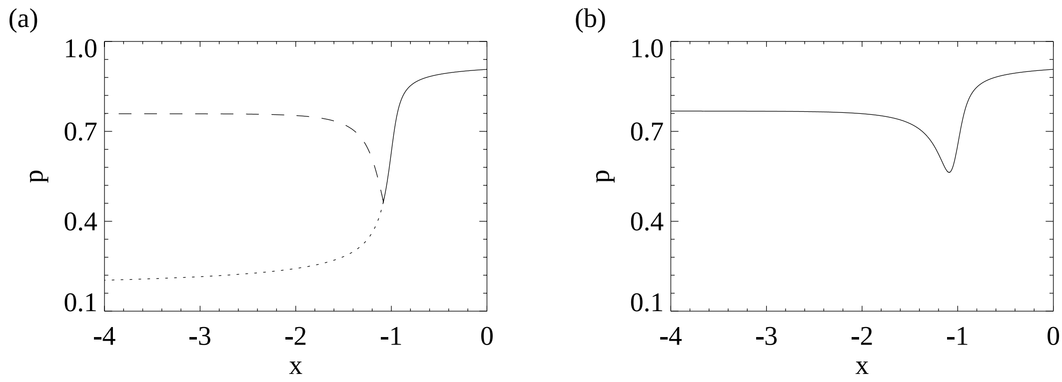

Figure 1: *(a) Pressure versus distance behind shock for the pathological detonation. The solid line is the branch of the solution between the shock and the sonic point, the dashed line is the branch between the sonic and strong equilibrium points (i.e. the supported pathological detonation) and the dotted line is the branch between the sonic and weak equilibrium points (i.e. the unsupported detonation). (b) Pressure versus distance behind shock for an overdriven detonation.*

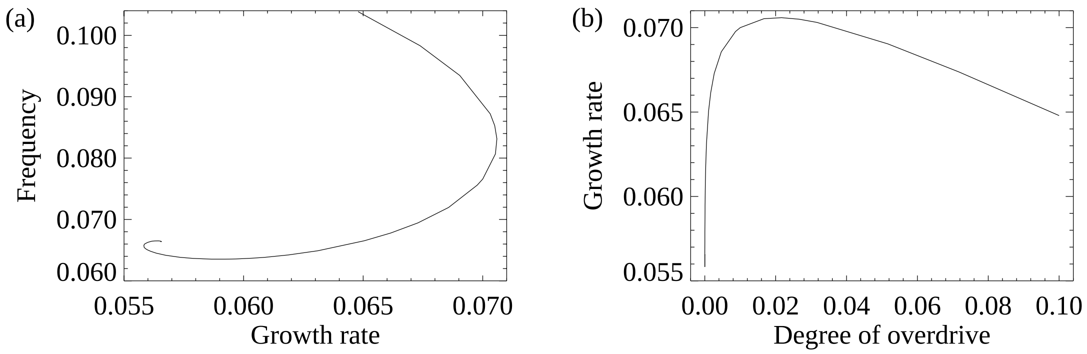

Figure 2: *Migration of a linear mode with degree of overdrive for one-dimensional disturbances.*

We investigate the stability of pathological detonations, and how this differs from systems where the unsupported detonation is the CJ wave. Our model system involves two consecutive irreversible reactions with the second reaction endothermic.

Stability of the underlying steady waves.

Firstly, due to the singular nature of the sonic pathological point, we develop asymptotic solutions of the governing equations for the steady, one-dimensional wave, valid near the sonic point in order to determine the structure of both the strong and weak solution branches. For example, figure 1 shows the pressure profiles for the unsupported and supported pathological detonations and an overdriven detonation for one particular case.

Secondly, we perform a linear stability analysis of the steady, one-dimensional pathological detonation. We show that the linear stability of the detonation to both one and two dimensional disturbances is very similar to that of CJ detonations. However, for overdriven detonations, we show that the predictions of the oscillation frequency for the pulsational instability, and of the cell size for the cellular instability are extremely sensitive to the detonation speed near the unsupported (pathological) detonation speed. For example, figure 2 shows the dependency of the fundamental mode on degree of overdrive for one-dimensional disturbances, with the other parameters held fixed. (Here we define the degree of overdrive by $(D^2 - D_p^2)/D_{CJ}^2$, where D is the detonation speed, D_p is the self-sustaining pathological speed and D_{CJ} is the Chapman-Jouguet reference speed). Such behaviour is markedly different than for systems where the unsupported wave is the CJ detonation.

Lastly, we perform numerical simulations of the pathological detonation, using an adaptive second order Godunov scheme, in order to investigate the long time, non-linear behaviour. Comparisons are made with the linear stability analysis, and differences between the evolution of the unsupported and supported waves are investigated. For example, figure 3 shows post-shock pressure histories for one-dimensional simulations. Figure 3(a) shows the pressure history for a self-sustaining detonation, which in this case is neutrally stable, while figures 3(b) and (c) show the pressure histories for corresponding overdriven detonations with degrees of overdrive 0.01 and 0.25 respectively. It can be seen that increasing the overdrive initially makes the detonation more unstable, but that for high enough overdrive the detonation is stable to one-dimensional perturbations, in agreement with the predictions of the linear stability analysis (cf figure 2).

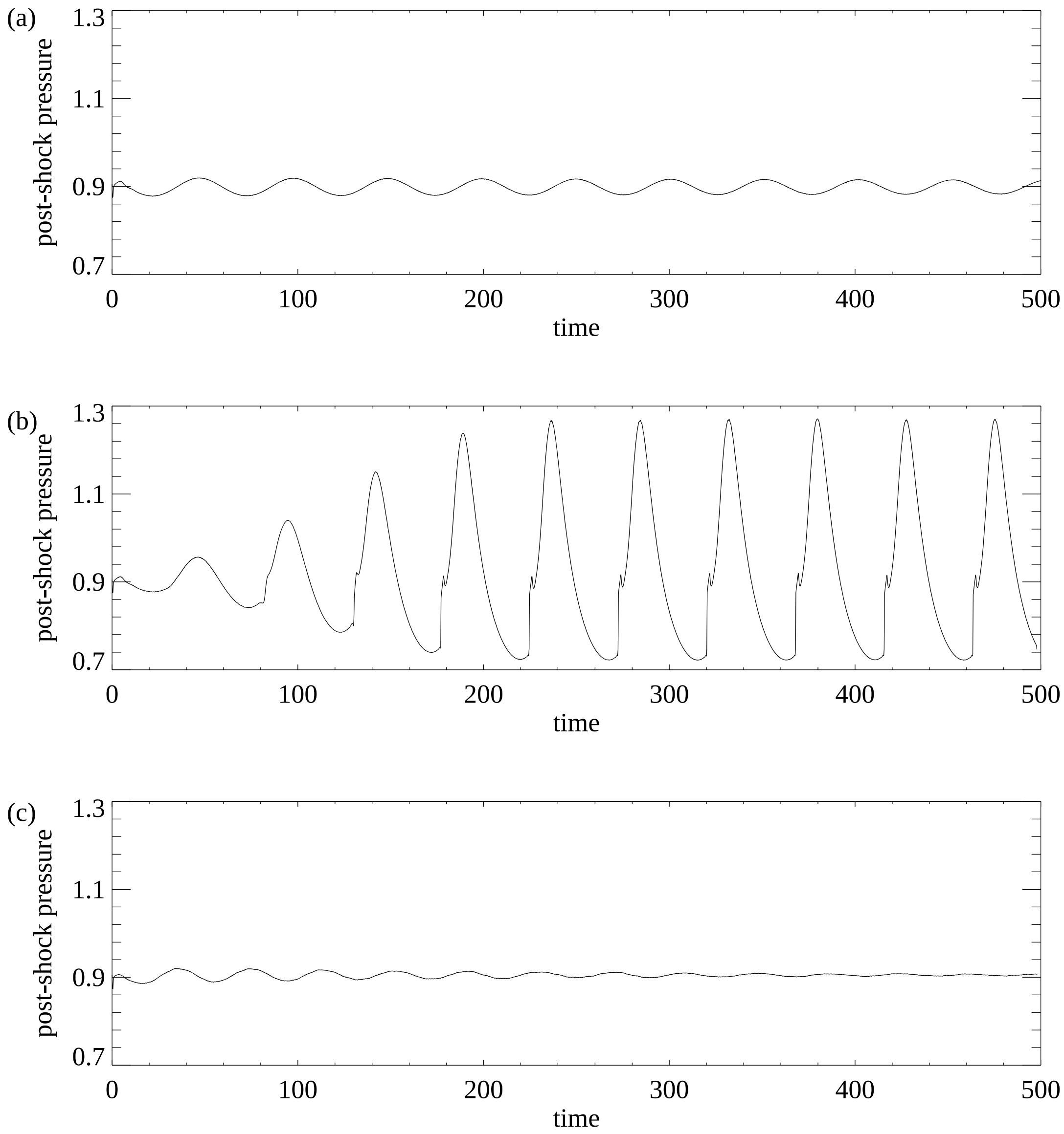

Figure 3: *Post-shock pressure histories from one-dimensional simulations for a particular case. (a) The self-sustaining pathological detonation and (b) and (c) overdriven detonations with degress of overdrive 0.01 and 0.25 respectively.*